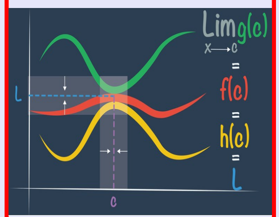
Math 261

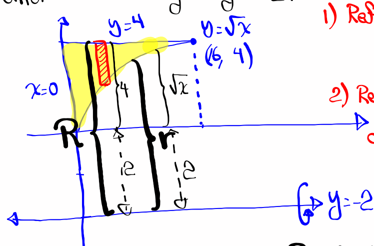
Spring 2023

Lecture 51



Feb 19-8:47 AM

Rotate the region bounded by $y=\sqrt{x}$, $y=4$, and $x=0$ by $y=-2$.



1) Ref. Rect. ⊥ A.O.R.
Yes

2) Region not completely attached to A.O.R.
Yes

Washer Method

$R = 4-(-2) = 6$ ✓

$r = \sqrt{x}-(-2) = \sqrt{x}+2$

$$V = \int_0^{16} \pi\left[R^2 - r^2\right]dx = \pi\int_0^{16}\left[6^2 - (\sqrt{x}+2)^2\right]dx$$

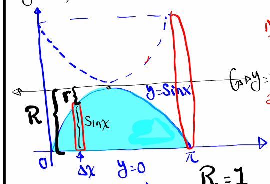
$$= \pi\int_0^{16}\left[36 - x - 4\sqrt{x} - 4\right]dx = \pi\int_0^{16}\left[32 - x - 4x^{1/2}\right]dx$$

$$= \pi\left(32x - \frac{x^2}{2} - \frac{4x^{3/2}}{3/2}\right)\Bigg|_0^{16} = \pi\left(32x - \frac{1}{2}x^2 - \frac{8}{3}x\sqrt{x}\right)\Bigg|_0^{16}$$

$$= \pi\left(32\cdot 16 - \frac{1}{2}\cdot 16^2 - \frac{8}{3}\cdot 16\sqrt{16}\right) = \boxed{\frac{640\pi}{3}}$$

May 16-9:28 AM

Rotate the region enclosed by $y = \sin x$, $y = 0$, in QI about $y = 1$. Find its volume.



1) Ref. Rect. ⊥ A.O.R. Yes

2) Region not Completely attached to A.O.R. Yes

Washer Method

$R = 1$ $\quad r + \sin x = 1$

$r = 1 - \sin x$

$$V = \pi\int_0^{\pi} (R^2 - r^2)\,dx = \pi\int_0^{\pi}\left[1 - (1 - \sin x)^2\right]dx$$

$$= \pi\int_0^{\pi}\left[\cancel{1} - \cancel{1} + 2\sin x - \sin^2 x\right]dx \qquad \sin^2 x \to \frac{1 - \cos 2x}{2}$$

$$= \pi\int_0^{\pi}\left[2\sin x - \frac{1 - \cos 2x}{2}\right]dx$$

$$= \pi\int_0^{\pi}\left[2\sin x - \frac{1}{2} + \frac{1}{2}\cos 2x\right]dx$$

$$= \pi\left[-2\cos x - \frac{1}{2}x\right]_0^{\pi} + \frac{\pi}{2}\int_0^{\pi}\cos 2x\,dx$$

$u = 2x$, $du = 2dx$, $\frac{du}{2} = dx$

$$= \pi\left[\left(-2\cancel{\cos\pi}^{-1} - \frac{\pi}{2}\right) - \left(-2\cancel{\cos 0}^{1} - \cancel{\tfrac{1}{2}\cdot 0}^{0}\right)\right]$$

$$\frac{\pi}{2}\int_0^{2\pi}\cos u \cdot \frac{du}{2}$$

$$= \pi\left[2 - \frac{\pi}{2} + 2\right]$$

$$= \pi\left(4 - \frac{\pi}{2}\right) + \frac{\pi}{4}\cancel{\left(\sin u\right)\Big]_0^{2\pi}}^{0} = \boxed{\frac{\pi(8 - \pi)}{2}}$$

May 16-9:38 AM

given $\quad f(x) = \frac{1}{6}(x^2 + 4)^{3/2}$

Find $\quad f'(x) = \frac{1}{\cancel{6}_2} \cdot \frac{\cancel{3}}{\cancel{2}}(x^2 + 4)^{\frac{3}{2} - 1} \cdot \cancel{2}x$

$$= \frac{1}{2}(x^2 + 4)^{1/2} \cdot x$$

Find $\quad 1 + [f'(x)]^2 = 1 + \left[\frac{x}{2}(x^2 + 4)^{1/2}\right]^2$

$$= 1 + \frac{x^2}{4}(x^2 + 4)$$

$$= \frac{x^4}{4} + x^2 + 1$$

finish this

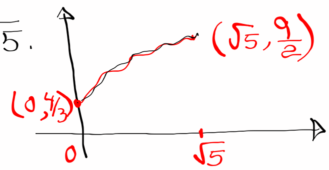
$$= \left(\frac{x^2}{2} + 1\right)^2$$

Now Find $\sqrt{1 + [f'(x)]^2} = \sqrt{\left(\frac{x^2}{2} + 1\right)^2}$

$$= \frac{x^2}{2} + 1$$

May 16-9:50 AM

Find the arc length along the curve given by $f(x) = \frac{1}{6}(x^2+4)^{3/2}$ from $x=0$ to $x=\sqrt{5}$.



$$L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx = \int_0^{\sqrt{5}} \left(\frac{x^2}{2}+1\right)dx$$

From last slide

$$= \left(\frac{1}{2}\cdot\frac{x^3}{3} + x\right)\Big]_0^{\sqrt{5}} = \frac{1}{6}(\sqrt{5})^3 + \sqrt{5} - 0$$

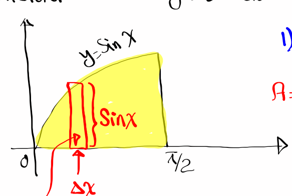
$$= \frac{1}{6}\cdot 5\sqrt{5} + 1\sqrt{5}$$

$$= \frac{5}{6}\sqrt{5} + \frac{6}{6}\sqrt{5}$$

$$= \left(\frac{5}{6}+\frac{6}{6}\right)\sqrt{5} = \boxed{\frac{11\sqrt{5}}{6}}$$

May 17-9:00 AM

Consider the region below:



$A_i = LW = \sin x_i \cdot \Delta x$

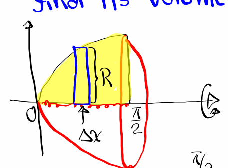
1) Find its area.

$$A = \int_0^{\frac{\pi}{2}} \sin x\,dx$$

$$= -\cos x\Big]_0^{\pi/2} = -\left[\cos\frac{\pi}{2} - \cos 0\right]$$

$$= -[0-1] = \boxed{1}$$

2) Rotate that region along x-axis, find its volume.



$R = \sin x$

1) Region is completely attached to A.O.R.

2) Rep. Rect. ⊥ A.O.R.

Disk method

$$V = \int_0^{\pi/2} \pi[\sin x]^2\,dx = \pi\int_0^{\pi/2} \sin^2 x\,dx$$

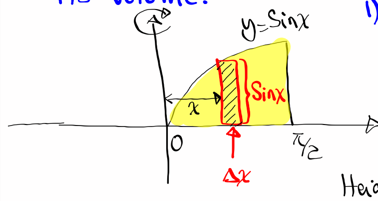
Recall from trig $\Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$

$$= \pi\int_0^{\pi/2} \frac{1-\cos 2x}{2}\,dx = \frac{\pi}{2}\int_0^{\pi/2}(1-\cos 2x)\,dx$$

Make Sure to finish this.

May 17-9:07 AM

3) Now rotate the region by Y-axis. Find its volume.



1) Is Ref. Rect. Parallel To A.O.R.? Yes

Shell Method

Height of Ref. Rec. ⇒ Sin x

Distance from A.O.R. ⇒ x

$$V = \int_a^b 2\pi \cdot \text{Distance} \cdot \text{Height}\, dx = \int_0^{\pi/2} 2\pi \cdot x \cdot \sin x\, dx$$

$$= 2\pi \int_0^{\pi/2} x \sin x\, dx$$

$$= 2\pi \Big[-x\cos x + \sin x \Big]_0^{\pi/2}$$

= Make Sure to finish this

Google integral of $x \sin x$

$$\int x \sin x\, dx = -x\cos x + \sin x$$

Verify

$$\frac{d}{dx}\left[-x\cos x + \sin x\right]$$

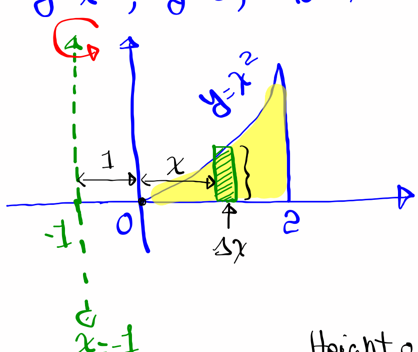
$$= -\left[1\cos x + x\cdot{}^{-}\sin x\right] + \cos x$$

$$= \cancel{-\cos x} + \boxed{x\sin x} \cancel{+\cos x}$$

May 17-9:19 AM

Consider the region bounded by

$y = x^2$, $y = 0$, and $x = 2$.



Rotate it about $x=-1$, Find its Volume.

Ref. Rect. is Parallel to A.O.R.

Shell Method

Height of Ref. Rect. → x^2

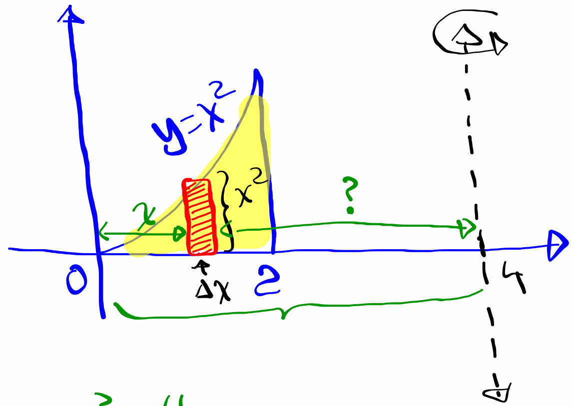
Distance from A.O.R. → $1+x$

$$V = \int_0^2 2\pi \cdot \text{Distance} \cdot \text{Height}\, dx = \int_0^2 2\pi \cdot (1+x) \cdot x^2\, dx$$

$$= 2\pi \int_0^2 (x^2 + x^3)\, dx =$$ Make Sure To finish

May 17-9:31 AM

Now rotate the same region by $x=4$.



Ref. Rect. Parallel to A.O.R.

Shell Method

Height of Ref. Rect. $\Rightarrow x^2$

Distance from A.O.R. $\Rightarrow 4-x$

$x + ? = 4$

$? = 4-x$

$$V=\int_0^2 2\pi \cdot \text{Distance} \cdot \text{Height}\, dx = \int_0^2 2\pi(4-x)\cdot x^2\, dx$$

Make sure to finish

May 17-9:40 AM

Find f_{ave} for $f(x)=\dfrac{x}{\sqrt{1+x^2}}$ on $[0,\sqrt{3}]$

$f(x)$ is cont. on $[0,\sqrt{3}]$

$$f_{ave}=\frac{1}{b-a}\int_a^b f(x)\,dx=\frac{1}{\sqrt{3}-0}\int_0^{\sqrt{3}}\frac{x}{\sqrt{1+x^2}}\,dx$$

$$=\frac{1}{2\sqrt{3}}\int_0^{\sqrt{3}}\frac{2x}{\sqrt{1+x^2}}\,dx$$

$u=1+x^2$

$du=2x\,dx$

$x=0 \rightarrow u=1$

$x=\sqrt{3} \rightarrow u=4$

$$=\frac{1}{2\sqrt{3}}\int_1^4\frac{1}{\sqrt{u}}\,du$$

$$=\frac{1}{2\sqrt{3}}\int_1^4 u^{-1/2}\,du=\frac{1}{\cancel{2}\sqrt{3}}\cdot\frac{u^{1/2}}{\cancel{1/2}}\Bigg]_1^4$$

$$=\frac{1}{\sqrt{3}}\left[\sqrt{u}\right]_1^4=\frac{1}{\sqrt{3}}\left(\sqrt{4}-\sqrt{1}\right)=\frac{1}{\sqrt{3}}=\boxed{\frac{\sqrt{3}}{3}}$$

May 17-9:46 AM